Hexile Sieve Analysis of Prime and Composite Integers.

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keywords : prime sieving, prime-counting function, diophantine equation, combinatorics.

Abstract

Here we demonstrate a sieve for analysing primes and their composites, using equivalence classes based on the modulo 6 return value as applied to the Natural numbers $\mathbb{N}$. Five features of this 'Hexile' sieve are reviewed. The first aspect, is that it narrows the search for primes to one-third of $\mathbb{N}$. The second feature is that we can obtain from the equivalence class formulae, a property of its diophantine equations to distinguish between primes and composites resulting from multiplication of these primes. Thirdly we can from these diophantine formulations ascribe a non-random occurence to not only the composites in the two equivalence classes but by default and as a consequence : non-randomness of occurence to the resident primes. Fourthly we develop a theoretical basis for sieving primes. Of final mention is that the diophantine equations allows another route to a prime counting function using combinatorics or numerical analysis.

1 Introduction to Hexile sieving of $\mathbb{N}$ using modulo 6.

It is not difficult to arrange $\mathbb{N}$ the set of Natural numbers into six equivalence classes, on the basis of their modulo 6 return value as shown overleaf in table 1.

Hexile integer-level : n	$\mathbb{H}^1$	$\mathbb{H}^2$	$\mathbb{H}^3$	$\mathbb{H}^4$	$\mathbb{H}^5$	$\mathbb{H}^0$
0	1	2	3	4	5	6
1	7	8	9	10	11	12
2	13	14	15	16	17	18
3	19	20	21	22	23	24
4	25	26	27	28	29	30
5	31	32	33	34	35	36
6	37	38	39	40	41	42
7	43	44	45	46	47	48
8	49	50	51	52	53	54
9	55	56	57	58	59	60
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Tab. 1: Hexile equivalence class sieving of the first 60 integers in $\mathbb{Z}^+$

Where the six equivalence classes are referenced as Hexile integer classes symbolized as $\mathbb{H}^{\mathrm{k}}$, where in set-builder notation we define the six classes thus for k $\in \{0, 1, 2, 3, 4, 5\}$:

$$\mathbb{H}^0 = [0] = \{6, 12, 18, ...\infty\} \equiv \left\{x | x = (6 \times n) + 0, n \in \mathbb{Z}^+, n \geq 0\right\} \tag{1.1}$$

$$\mathbb{H}^1 = [1] = \{7, 13, 19, ...\infty\} \equiv \left\{x | x = (6 \times n) + 1, n \in \mathbb{Z}^+, n \geq 0\right\} \tag{1.2}$$

$$\mathbb{H}^2 = [2] = \{8, 14, 20, ...\infty\} \equiv \left\{x | x = (6 \times n) + 2, n \in \mathbb{Z}^+, n \geq 0\right\} \tag{1.3}$$

$$\mathbb{H}^3 = [3] = \{9, 15, 21, ...\infty\} \equiv \left\{x | x = (6 \times n) + 3, n \in \mathbb{Z}^+, n \geq 0\right\} \tag{1.4}$$

$$\mathbb{H}^4 = [4] = \{10, 16, 22, ...\infty\} \equiv \left\{x | x = (6 \times n) + 4, n \in \mathbb{Z}^+, n \geq 0\right\} \tag{1.5}$$

$$\mathbb{H}^5 = [5] = \{11, 17, 23, ...\infty\} \equiv \left\{x | x = (6 \times n) + 5, n \in \mathbb{Z}^+, n \geq 0\right\} \tag{1.6}$$

Closer examination of table 1, reveals that $\mathbb{H}^0$, $\mathbb{H}^2$ and $\mathbb{H}^4$ contains all the even numbers, consequently with the exception of 2 in $\mathbb{H}^2$, they are all composites which are multiples of 2. We will also observe that $\mathbb{H}^3$ contains only one element which is prime: 3, the other elements being multiples of 3.

Which leaves us with $\mathbb{H}^1$ and $\mathbb{H}^5$ as candidates for the domiciled sub-set partitioning of primes in $\mathbb{N}$. Within which we will find all primes greater than 3, as well as composites which are themselves product of these primes.

Hence prime numbers greater than or equal to 5, can be found in either of the two HICs :

$\mathbb{H}^1$ and $\mathbb{H}^5$ defined above by equations 1.2 and 1.6. [1]

[1] HIC is an abbreviation for Hexile integer class(es).

2 Formulae of composites in $\mathbb{H}^1$ and $\mathbb{H}^5$.

We will now examine the nature of composites in $\mathbb{H}^1$ *and* $\mathbb{H}^5$, which also details the autochtnous nature of their multiplicative origin from endemic primes.

2.1 Composite resulting from multiplying two primes from $\mathbb{H}^1$.

If we were to consider two primes $p_1, q_1 \in \mathbb{H}^1$ with formulation $p_1 = (6 \times m) + 1$ and $q_1 = (6 \times n) + 1$ where $m, n \in \mathbb{Z}^+$.

Multiplication of primes p_1 and q_1 yields a composite c^{11} whence :

$$\begin{aligned} c^{11} &= p_1 \times q_1 \\ &= [(6 \times m) + 1] \times [(6 \times n) + 1] \\ \therefore c^{11} &= (36 \times m \times n) + (6 \times m) + (6 \times n) + 1 \end{aligned} \tag{2.1}$$

Applying modulo 6 to the result of equation 2.1 returns a value of 1, it then logically implies that the composite : c^{11} is an element of $\mathbb{H}^1$.

2.2 Composite resulting from multiplying two primes from $\mathbb{H}^5$.

If we were to consider two primes $p_5, q_5 \in \mathbb{H}^5$ with formulations $p_5 = (6 \times m) + 5$ and $q_5 = (6 \times n) + 5$ where $m, n \in \mathbb{Z}^+$.

Multiplication of primes p_5 and q_5 yields a composite c^{55} whence :

$$\begin{aligned} c^{55} &= p_5 \times q_5 \\ &= [(6 \times m) + 5] \times [(6 \times n) + 5] \\ \therefore c^{55} &= (36 \times m \times n) + (30 \times m) + (30 \times n) + 25 \end{aligned} \tag{2.2}$$

Applying modulo 6 to the result of equation 2.2 returns a value of 1, it then logically implies that the composite : c^{55} is an element of $\mathbb{H}^1$.

2.3 Composite resulting from multiplying one prime from $\mathbb{H}^1$ with another prime from $\mathbb{H}^5$.

If we were to consider two primes $p_1 \in \mathbb{H}^1$ with formulation $p_1 = (6 \times m) + 1$ and $q_5 \in \mathbb{H}^5$ with formulation $q_5 = (6 \times n) + 5$ where $m, n \in \mathbb{Z}^+$.

Multiplication of primes p_1 and q_5 yields a composite c^{15} whence :

$$c^{15} = p_1 \times q_5 = [(6 \times m) + 1] \times [(6 \times n) + 5]$$

$$\therefore c^{15} = (36 \times m \times n) + (30 \times m) + (5 \times n) + 5 \tag{2.3}$$

Applying modulo 6 to the result of equation 2.3 returns a value of 5, it then logically implies that the composite : c^{15} is an element of $\mathbb{H}^5$.

2.4 Possibility of $\mathbb{H}^1$ and $\mathbb{H}^5$composites arising from multiplication of other HIC elements.

Elements from $\mathbb{H}^0$, $\mathbb{H}^2$ and $\mathbb{H}^4$ are all even numbers, since multiplying an even number with an odd number always gives an even number, then their multiplication cannot possibly give rise to $\mathbb{H}^1$ and $\mathbb{H}^5$ composites.

We are left then with the sole possibility of an element of $\mathbb{H}^3$, giving rise to a composite in $\mathbb{H}^1$ and $\mathbb{H}^5$. Then for $r_3 \in \mathbb{H}^3$, with a generic formulation of :

$$r_3 = (6 \times d) + 3 \tag{2.4}$$

We next need to examine the end-result of multiplying r_3 with an element of $\mathbb{H}^1$, $\mathbb{H}^3$ and $\mathbb{H}^5$. Table 2 below, shows the results of these three combinations, where $p_1 \in \mathbb{H}^1$ and $q_5 \in \mathbb{H}^5$ are formulated thus:

$$p_1 = (6 \times m) + 1 \tag{2.5}$$

$$q_5 = (6 \times n) + 5 \tag{2.6}$$

	$r_3 = (6 \times d) + 3$	modulo 6 of composite
r_3	$c^{33} = \left(36 \times d^2\right) + (36 \times d) + 9$	3
p_1	$c^{13} = (36 \times d \times m) + (18 \times m) + (6 \times d) + 3$	3
q_5	$c^{35} = (36 \times d \times n) + (30 \times d) + (18 \times n) + 15$	3

Tab. 2: Results of multiplying $r_3 \in \mathbb{H}^3$ with an element of $\mathbb{H}^1$, $\mathbb{H}^3$ and $\mathbb{H}^5$.

The results from table 2, indicates an element of $\mathbb{H}^3$ will not result in a $\mathbb{H}^1$ or $\mathbb{H}^5$ composite through multiplication. So then composites in $\mathbb{H}^1$ and $\mathbb{H}^5$

are found autochthonous with primes from which they are the product through multiplication.

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3 Investigating the formulation of composites, through the contribution of the Hexile Levels of their constituent primes.

Our probings will now take us to the inner sanctum of the formulaic expression of composites, into - for want of a more apt term, we will refer to as - the *nucleus* of an element.

3.0.1 Definition of the *nucleus* value of a Natural number.

We define the *nucleus* value of any Natural number, as the quotient obtained when that number is divided by 6.

Of note is that the *nucleus* value can also be referred to as the Hexile level of a Natural number, which was initially referenced as the row numbers in the Hexile Sieve table 1 of section (1).

Applying the above definition (3.0.1) to the three formulations of $\mathbb{H}^1$ and $\mathbb{H}^5$ composites obtained in the previous sub-sections (2.1), (2.2) and (2.3), we obtain the following three formulations for *nuclei* values as shown in table 3.

Composite Formulae	Nucleus Formulae
$c^{11} = (36 \times m \times n) + (6 \times m) + (6 \times n) + 1$	$(6 \times m \times n) + m + n$
$c^{55} = (36 \times m \times n) + (30 \times m) + (30 \times n) + 25$	$(6 \times m \times n) + (5 \times m) + (5 \times n) + 4$
$c^{15} = (36 \times m \times n) + (30 \times m) + (5 \times n) + 5$	$(6 \times m \times n) + (5 \times m) + n$

Tab. 3: Showing nucleus value formulations for $\mathbb{H}^1$ and $\mathbb{H}^5$ composites.

The results of table 3 can be used as a preliminary investigative formulative-decomposition tool, for an element of $\mathbb{H}^1$ and $\mathbb{H}^5$.

Whereby if we are given a positive integer c, if its modulo 6 value returned is 1 and its easily obtained *nucleus* value is Q, then we can ascribe to it as a composite one of the following two formulations of equations 3.1 and 3.2 given below :

$$Q = (6 \times m \times n) + m + n \tag{3.1}$$

or

$$Q = (6 \times m \times n) + (5 \times m) + (5 \times n) + 4 \tag{3.2}$$

Also for a given Natural number c, if its modulo 6 value returned is 5 and its *nucleus* value is Q, we can specifically ascribe to it as a composite the formulation of equation 3.3 given below:

$$Q = (6 \times m \times n) + (5 \times m) + n \tag{3.3}$$

Where $m, n \in \mathbb{Z}^+$ are the Hexile levels of the two constituent integers presumed to be primes in the composite elements of $\mathbb{H}^1$ and $\mathbb{H}^5$ as symbolized by the variable c.

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4 Distinguishing feature of Composite linear diophantine equations for elements of $\mathbb{H}^1$ and $\mathbb{H}^5$.

We investigated in the previous section (3) three diophantine formulations for composites in $\mathbb{H}^1$ and $\mathbb{H}^5$.

These three equations viz : (3.1), (3.2) and (3.3), are essentially formulae defining the relationship of a composite's *nucleus* value in terms of the Hexile levels of its two constituent primes/composites.

We will for ease of referential paltabilty designate equations (3.1), (3.2) and (3.3) as *Composite Linear Diophantine Equation*(s), or *colide(s)* for short.

Specifically we will reference a colide-11 to equation 3.3 viz. :

$$Q = (6 \times m \times n) + m + n \qquad 3.3$$

where m, n are the **HI-level**s of the two constituent primes/composites with $m, n \in \mathbb{Z}^+$, $m, n > 0$, with Q being the readily computable *nucleus* value.

Also a colide-55, we will reference to equation 3.2 below :

$$Q = (6 \times m \times n) + (5 \times m) + (5 \times n) + 4 \qquad 3.2$$

where m, n are the **HI-level**s of the two constituent primes/composites with $m, n \in \mathbb{Z}^+$, $m, n \geqq 0$ with Q being the computable *nucleus* value.

And a colide-15 we will refer to by equation 3.3 below :

$$Q = (6 \times m \times n) + (5 \times m) + n \qquad 3.3$$

where m, n are the **HI-level**s of the two constituent primes/composites with $m, n \in \mathbb{Z}^+$, $m > 0$, $n \geqq 0$ with Q being the readily computable *nucleus* value.

The importance of these three colides is that *they have no integer solutions* for primes, else if they possibly do, by contradiction they are composite as we can derive back m and n which will be the Hexile levels of occurence of its two multiplicand integers.

Using these equations in the following section 5, we will attempt to examine the sequencing of *nuclei* values in these colides : which of course only references the Hexile level of composites.

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5 Sequencing of Nuclei values for $\mathbb{H}^1$ and $\mathbb{H}^5$.

In the following sub-sections, the sequencing of composites in $\mathbb{H}^1$ and $\mathbb{H}^5$ is reviewed for the obvious non-random nature of its series.

5.1 Evaluating composite occurrences for a Colide-11 composite.

Recalling a colide-11 viz. :

$$Q = (6 \times m \times n) + m + n \tag{3.1}$$

where $m, n \in \mathbb{Z}^+$ are the **HI-level**s of the two constituent primes/composites, with Q being the readily computable *nucleus* value.

We can transmute equation (3.1) into the following function :

$$f_{11}(m, n) = [(6 \times m \times n)] + m + n \tag{5.1}$$

this function taking on unordered integer pairs : (m, n) of the HI-levels of the two constituent primes/composites, where $m, n \in \mathbb{Z}^+, \; m, n > 0$.

We will designate this function as the c̲olide-11 s̲tate function, abbreviated to CS-11 function.

Table 4 shows the values obtained by applying to the CS-11 function the first 8 positive integers excluding zero. The resulting symmetric matrix, with its main diagonal is highlighted in bold.

LEVEL : (m) :	1	2	3	4	5	6	7	8
$f_{11}\,(m,\, n\,) = [(6 \times m \times n)] + m + n$	c_N^{11}	c_N^{11}	c_N^{11}	c_N^{11}	c_N^{11}	c_N^{11}	c_N^{11}	c_N^{11}
LEVEL : (n) :								
n = 1	***8***	*15*	*22*	*29*	*36*	*43*	*50*	*57*
n = 2	*15*	***28***	*41*	*54*	*67*	*80*	*93*	*106*
n = 3	*22*	*41*	***60***	*79*	*98*	*117*	*136*	*155*
n = 4	*29*	*54*	*79*	***104***	*129*	*154*	*179*	*204*
n = 5	*36*	*67*	*98*	*129*	***160***	*191*	*222*	*253*
n = 6	*43*	*80*	*117*	*154*	*191*	***228***	*265*	*302*
n = 7	*50*	*93*	*136*	*179*	*222*	*265*	***308***	*351*
n = 8	*57*	*106*	*155*	*204*	*253*	*302*	*351*	***400***
...	...	...	...	...	...	...	...	...

Tab. 4: Table showing the resulting 8×8 - symmetric matrix of colide-11 values obtained on applying the CS-11 function to the various combination-pairs of the first 8 positive integers excluding zero.

The resulting tabulation of values in Table 4 is a symmetric matrix, showing the predictable sequencing of nucleus values for a colide-11 composite.. This symmetry is due to the commutative-mutable-emplacement of the m and n variables, within the CS-11 function thus :

$$f_{11}\,(m,\, n\,) = (6 \times m \times n) + m + n \tag{5.1}$$

$$i.e.\ f_{11}\,(m,\, n\,) = [(6 \times m) + 1] \times n + m = [(6 \times n) + 1] \times m + n \tag{5.2}$$

The above equation 5.2 of a CS-11 function, thus governs the sequencing of colide-11 Hexile levels - with an arithmetic progression, hence producing a series whose sequence is predictable and non-random in nature.

Of critical note, is that the main diagonal of the symmetric matrix in Table 4 represents those instances where $m = n$ of our CS-11 function.
To which, applying values of $m = n$ to equation 5.1 will confirm as computationally defined by:

$$f_{11}\,(m,\, m\,) = \left(6 \times m^2\right) + (2 \times m) \tag{5.3}$$

5.2 Evaluating composite occurrences for a Colide-55 composite.

We met in the section 5 the equation for a colide-55 viz.:

$$Q = (6 \times m \times n) + (5 \times m) + (5 \times n) + 4 \tag{3.2}$$

where $m,\, n \in \mathbb{Z}^+$ are the HI-levels of the two constituent primes/composites, with Q being the readily computable *nucleus* value.

Whereby equation (3.2) can be transmuted to the function :

$$f_{55}\,(\,m,\,n\,) \;=(6 \times m \times n) + \;(5 \times m) + \;(5 \times n) \;+\; 4 \tag{5.4}$$

this function taking on unordered integer pairs : $(\,m,\,n\,)$ of the HI-levels of the two constituent primes/composites, where $m,\, n \,\in\, \mathbb{Z}^+$, with $m,\, n \,\geqq\, 0$.

We will designate this function as the colide-55 state function, abbreviated to CS-55 function.

Table 5 shows the values obtained by applying to the CS-55 function the first 8 positive integers excluding zero.The resulting symmetric matrix, with its main diagonal is highlighted in bold.

LEVEL : (m) :	0	1	2	3	4	5	6
$f_{55}\,(m,n) = (6 \times m \times n) + (5 \times m) + (5 \times n) + 4$	c_N^{55}	c_N^{55}	c_N^{55}	c_N^{55}	c_N^{55}	c_N^{55}	c_N^{55}
LEVEL : (n) :							
n = 0	**4**	9	14	19	24	29	34
n = 1	9	**20**	*31*	*42*	53	64	75
n = 2	14	*31*	***28***	*41*	*54*	*67*	*80*
n = 3	19	*42*	*41*	***60***	*79*	*98*	*117*
n = 4	24	53	*54*	*79*	***104***	*129*	*154*
n = 5	29	*64*	*67*	*98*	*129*	***160***	*191*
n = 6	34	75	*80*	*117*	*154*	*191*	***228***
...		...	...	...	...	...	...

Tab. 5: Table showing the resulting 7×7 - symmetric matrix of colide-55 values obtained on applying the CS-55 function to the various combination-pairs of the first 7 positive integers including zero.

The resulting tabulation of values in Table 5 is a symmetric matrix, showing the predictable sequencing of nucleus values for a colide- 55 composite. This symmetry is due to the commutative-mutable-emplacement of the m and n variables, within the CS-55 function thus :

$$f_{55}\,(\,m,\,n\,) = (6 \times m \times n) + (5 \times m) + (5 \times n) + \;4 \tag{5.4}$$

$$i.e.\; f_{55}\,(\,m,\,n\,) = [(6 \times m) \;+\; 5]{\times}n{+}(5 \times m) \;+\; 4 \;=\; [(6 \times n) \;+\; 5] \times m + \;(5 \times n) + \;4 \tag{5.5}$$

The above equation 5.5 of a CS-55 function, thus governs the sequencing of colide-55 Hexile levels - with an arithmetic progression, hence producing a series whose sequence is predictable and non-random in nature..

Also of critical note is that the main diagonal of the symmetric matrix in Table 5 represents those instances where $m = n$ of our **CS-55** function.
To which, applying values of $m = n$ to equation 5.4 will confirm as computationally defined by equation 5.6 below:

$$f_{55}\,(m,\, m) \;=\; \left(6 \,\times\, m^2\right) +\; (10 \,\times\, m) \;+\; 4 \tag{5.6}$$

5.3 Evaluating composite occurrences for a Colide-15 composite.

Looking at the equation for a colide-15 from section 5 to wit:

$$Q \;= (6 \times m \times n) + \;(5 \times m) \;+\; n \tag{3.3}$$

Whereby equation (3.3) can be transmuted to the function :

$$f_{15}\,(m,\, n) \;= (6 \times m \times n) + \;(5 \times m) + \; n \tag{5.7}$$

this function taking on ordered integer pairs : $(m,\, n)$ of the HI-levels of the two constituent primes/composites, where $m,\, n \,\in\, \mathbb{Z}^+$, with $m,\, n \,\geqq\, 0$.

We will designate this function as the colide-15 state function, abbreviated to CS-15 function.

Also of critical note is that the input values are ordered pairs $(m,\, n)$, and that they do not necessarily represent the HI-levels of primes/composites.

Table 6 shows the values obtained by applying to the CS-55 function the first 8 positive integers excluding zero.

LEVEL : (m) :	1	2	3	4	5	6	7	8
$f_{15}\,(m,n) = (6 \times m \times n) + (5 \times m) + n$	c_N^{15}	c_N^{15}	c_N^{15}	c_N^{15}	c_N^{15}	c_N^{15}	c_N^{15}	c_N^{15}
LEVEL : (n) :								
n = 1	12	23	34	45	56	67	78	89
n = 2	19	36	53	70	87	104	121	138
n = 3	26	49	72	95	118	141	164	187
n = 4	33	62	91	120	149	178	207	236
n = 5	40	75	110	145	180	215	250	285
n = 6	47	88	129	170	211	252	293	334
n = 7	54	101	148	195	242	289	336	383
n = 8	61	114	167	220	273	326	379	432
...		...	...	...	...	...	...	...

Tab. 6: The resultant 8×8 - matrix table, showing the colide-15 values obtained on applying the CS-15 function to various combination-pairs of the first 8 positive integers excluding zero.

The resulting tabulation of values in Table 5 shows the predictable sequencing of nucleus values for a colide-15 composite.

$$f_{15}\,(m,\,n\,)\;=(6\times m\times n)+\;(5\times m)+\;n \tag{5.7}$$

$$i.e.\; f_{11}\,(m,\,n\,)\;=[(6\times m)\;+\;1]\times n+(5\times m)\;=\;[(6\times n)\;+\;5]\times m+\;n \tag{5.8}$$

From equation 5.8 it is obvious that the CS-15 function sequences *nuclei* values is an arithmetic progression, hence producing a series whose sequence is predictable and non-random in nature.

5.4 Conclusion to Sequencing of Nuclei values for $\mathbb{H}^1$ and $\mathbb{H}^5$ elements.

In the preceding sub-sections (5.1) and (5.2), we were able to establish that colide-11 and colide-55 composites which populate $\mathbb{H}^1$, occur in a non-random sequence. Then from the result in section (4) where we assayed $\mathbb{H}^1$ to consist of only colide-11 and colide-55 composites and primes, this means we can logically deduce : that primes which are the only other elements of $\mathbb{H}^1$, occur in a non-random fashion in the vacant Hexile levels not occupied by colide-11s and colide-55s.

With similiar reasoning from the conclusion drawn in sub-section (5.3) we found that the colide-15 composites which populate $\mathbb{H}^5$ also occur in a non-random sequence within this equivalence class. Then from the result in section (4) where we assayed $\mathbb{H}^5$ to consist of only colide-15 composites and primes, this also means we can logically deduce : that primes which are the only other elements found in $\mathbb{H}^5$, also occur in a non-random fashion in the vacant Hexile levels not occupied by colide-15s.

The conclusion can then be drawn that we have crudely established the rhythm and harmonics in the symphony of prime and composite integers.

As a preliminary to the next section 6, where the cardinality of primes in $\mathbb{H}^1$ and $\mathbb{H}^5$ is examined, we examine the following prime sieve. We introduce this, as the over-arching theoretical underpinnings of being able to sieve primes, also bears directly on being able to count them.

We found a way to generate the sequence of integers representing the *Hexile levels* of composites populating $\mathbb{H}^1$ using the CS-11 and CS-55 functions. This means we can conceptualise and designate this series of integers as a set symbolized by S. Then the set T, which is the set representing the *Hexile levels* of primes in $\mathbb{H}^1$ can be naively found by *:*

$$\mathrm{T} = \mathbb{N} - \mathrm{S} \tag{5.9}$$

Similiarly for primes in $\mathbb{H}^5$ *we can* generate the set of integers representing composite Hexile levels using the CS-15 function and designate this set by V.

Then the set W which is the set representing the Hexile levels of primes in $\mathbb{H}^5$ can also naively be found by *:*

$$\mathrm{W} = \mathbb{N} - \mathrm{V} \tag{5.10}$$

6 Counting primes in $\mathbb{H}^1$ *and* $\mathbb{H}^5$.

With the three colides serving as the theoretical basis for composites, we have gained a little insight into the multiplicative mechanics of composite generation.[2]

Then it should be easy for us to find a process whereby and wherein for *any* given positive integer in $\mathbb{N}$, to undertake the following steps in computing the number of Hexile levels occupied by primes less than that integer :

1. Obtaining the nucleus value Q for a given integer c.
2. Obtaining maximal Hexile level values which satisfies the three colide(s) formulations viz:

   (a) The maximal Hexile level value of a colide-11, being the floor integer which satisfies the colide-11, when one of its variables is equal to 1, in the case of $m = 1$ we get:

   $$\begin{aligned} Q &\geq (6 \times m \times n) + m + n \\ &\geq (6 \times 1 \times n) + 1 + n \\ \therefore\ Q &\geq (7 \times n) + 1 \end{aligned} \tag{6.1}$$

   Then n_{11} the maximal Hexile Level value satisfying equation (6.1) for a colide-11 is computed from:

   $$n_{11} = floor\left[\frac{Q - 1}{7}\right] \tag{6.2}$$

   where " $floor$ [ ] ", refers to the integer floor function, with $n_{11} \in \mathbb{Z}^+$.

   (b) The maximal Hexile level value of a colide-55 being the floor integer which satisfies the colide-55, when one of its variables is equal to 1, in the case of $m = 1$ we get:

   $$\begin{aligned} Q &\geq (6 \times m \times n) + (5 \times m) + (5 \times n) + 4 \\ &\geq (6 \times 1 \times n) + 5 + (5 \times n) + 4 \end{aligned}$$

   $$\therefore\ Q \geq [(11 \times n)] + 9 \tag{6.3}$$

   Then n_{55}, the maximal Hexile Level value satisfying equation (6.3) for a colide-55 is computed from :

   $$n_{55} = floor\left[\frac{Q - 9}{11}\right] \tag{6.4}$$

   where " $floor$ [ ] ", refers to the integer floor function, with $n_{55} \in \mathbb{Z}^+$.

[2] Not all composites in $\mathbb{N}$ are in our present consideration, but composites which are important to asymmetric encryption in the PKI industry: as found in $\mathbb{H}^1$ and $\mathbb{H}^5$. For composites in equivalence classes : $\mathbb{H}^0$, $\mathbb{H}^2$, $\mathbb{H}^3$ and $\mathbb{H}^4$ were not explored, as these HICs do not contain primes greater than 5, and less importantly their composites are easily decomposed into multiples of either 2 or 3.

(c) The maximal Hexile level value of a colide-15, being the floor integer which satisfies the colide-15 when one of its variables is equal to 1, in the case of $m = 1$ we get :

$$Q \geq (6 \times m \times n) + (5 \times m) + n$$

$$\geq [(6 \times 1 \times n)] + 5 + n$$
$$\therefore Q \geq [(7 \times n)] + 5 \tag{6.5}$$

Then n_{15} the maximal Hexile Level value satisfying equation equation equation 6.5 for a colide-55 is computed from :

$$n_{15} = floor\left[\frac{Q - 5}{7}\right] \tag{6.6}$$

where " $floor[\ ]$ ", refers to the integer floor function, with $n_{15} \in \mathbb{Z}^+$

3. The next stage is computing the number of composites below the given integer c. It is here we can apply combinatorial or numerical analysis for determining the *number* of integer tuples (m, n) satisfying the following equalities within the domain of integers from 1 to their maximal Hexile Level(s) for the three 'flavours' of colides.

(a) Where for the following colide-11 equation 6.7 below :

$$Q \geq (6 \times m \times n) + m + n \tag{6.7}$$

we compute the *number* of integer tuples (m, n) within the domain of integers in the set $\{1, ..., n_{11}\}$ which satisfies the above equation 6.7.
Noting that we have to take into account instances where there are *duplicated Hexile level values*. This resulting from instances where the values of tuples (m, n) are interchanged, yielding the same Hexile level value in the CS-11 function. This occurs through the commutatively emplaced variables as shown in equation (5.2) below :

$$f_{11}(m, n) = [(6 \times m) + 1] \times n + m = [(6 \times n) + 1] \times m + n \tag{5.2}$$

Additionally we are to take into account *none-duplicated Hexile level values*. This resulting from instances where the values of tuples (m, n) are identical. This was noted before at the end of sub-section (5.1) as accounting for the symmetric matrix of graphed CS-11 values as shown in equation (5.3) repeated below :

$$f_{11}(m, m) = \left(6 \times m^2\right) + (2 \times m) \tag{5.3}$$

(b) Where for the following colide-55 equation 6.8 below :

$$Q \geq (6 \times m \times n) + (5 \times m) + (5 \times n) + 4 \tag{6.8}$$

we compute the *number* of integer tuples $(m,\, n)$ within the domain of integers in the set $\{1, ..., n_{55}\}$ which satisfies the above equation 6.8.
Noting that we have to take into account instances where there are *duplicated Hexile level values*. This resulting from instances where the values of tuples $(m,\, n)$ are interchanged, yielding the same Hexile level value in the CS-55 function. This occurs through the commutatively emplaced variables as shown in equation (5.5) below :

$$\begin{aligned} f_{55}(m, n) &= [(6 \times m) + 5] \times n + (5 \times m) + 4 \\ &= [(6 \times n) + 5] \times m + (5 \times n) + 4 \end{aligned} \tag{5.5}$$

Additionally we are to take into account *none-duplicated Hexile level values*. This resulting from instances where the values of tuples $(m,\, n)$ are identical. This was noted before as accounting for the symmetric matrix of graphed CS-55 values at the end of sub-section (5.2) in equation (5.6) repeated below :

$$f_{55}(m,\, m) = \left(6 \times m^2\right) + (10 \times m) + 4 \tag{5.6}$$

and

(c) Where for a colide-15 equation 6.9 below :

$$Q \geq (6 \times m \times n) + (5 \times m) + n \tag{6.9}$$

we compute the *number* of integer tuples $(m,\, n)$ within the domain of integers in the set $\{1, ..., n_{15}\}$ which satisfies the above equation 6.9.

4. The number of primes can then be directly computed as the difference of twice the nucleus value Q, less the total number of computed composites as determined from the total number of 'viable' integer tuples $(m,\, n)$ computed in the previous step.
We of course have to add into this figure the first three primes.

In a general sense the essential logic in the above algorithm, is predicated primarily on being able to precisely compute the total number of $\mathbb{H}^1$ and $\mathbb{H}^5$ composites below any positive integer. This together with being able to determine the total number of elements in $\mathbb{H}^1$ and $\mathbb{H}^5$ (as being twice the nucleus value Q). Then we can determine by default and deduction both logically and computationally with a high degree of assurance, the total number of primes below that given integer.

7 Conclusion to the Hexile Sieve Analysis of primes and composites.

Even though our discussion is a basic summary of some of the areas researched, three objectives were intended in this expression, namely :

1. A simple explanation for critical mysteries surrounding primes and composites.
2. A viable alternative observational and theoretical basis for re-examining how Natural numbers are conceptualised.
3. And hopefully a stimulus and starting point to solving other key areas of conjectures and interest in Number Theory.

As for succesful intent and impact by these three aspirations into prime number theory, and criticism into the overall and/or specific logic and flaws in this discourse, the author welcomes and entertains feedback.

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email : hexilehelix@yahoo.com